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# Universal conductance in quantum wires in the presence of Umklapp scattering*

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**Abstract.** The effects of Umklapp scattering on the zero-temperature conductance in one-dimensional quantum wires are reexamined by taking into account both the screening of external potential and the non-uniform chemical potential shift due to electron-electron interaction. It is shown that in the case away from half-filling the conductance is given by the universal value, $2e^2/h$, even in the presence of Umklapp scattering, owing to these renormalization effects of external potential. The conclusion is in accordance with the recent claim obtained for the system with non-interacting leads being attached to a quantum wire.

**PACS.** 71.10.Pm Fermions in reduced dimensions (anyons, composite fermions, Luttinger liquid, *etc.*) – 72.15.Nj Collective modes (*e.g.*, in one-dimensional conductors) – 72.10.Fk Scattering by point defects, dislocations, surfaces, and other imperfections (including Kondo effect)

## 1 Introduction

The effects of electron-electron interaction on the conductance in one-dimensional (1D) quantum wires have been extensively studied from both theoretical and experimental points of view. The low-energy properties of 1D interacting electron systems are described by the Tomonaga-Luttinger (TL) liquids [1]. It has been known that the conductance at zero temperature is given by $2e^2K_\rho/h$, where $K_\rho$ is the TL liquid parameter which controls the asymptotic behavior of correlation functions [2,3]. However, according to the recent experiment, the observed conductance is not $2e^2K_\rho/h$ but $2e^2/h$, which is expected for the conductance of 1D non-interacting electron systems [4]. In order to explain this discrepancy, two possible scenarios have been proposed. One is that non-interacting leads attached to a quantum wire are essential to reproduce the observed conductance $2e^2/h$ [5–7]. The other scenario is that if one takes into account the screening of external potential due to electron-electron interaction, the multiplicative factor $K_\rho$ in the conductance may become unity [8–10]. Both of these scenarios give the nice explanations for the experimentally observed conductance, if only forward scattering of electron-electron interaction exists. However, in the presence of Umklapp or impurity scattering which gives rise to momentum dissipation, these scenarios lead to different results [9,11–19]. In the case that non-interacting leads are attached, the conductance is still given by the unrenormalized value, $2e^2/h$, even in the presence of Umklapp scattering as far as the electron density is away from half-filling [17,18]. In contrast, if one takes into account only the screening of external potential due to electron-electron interaction, the Umklapp scattering gives the non-universal value of conductance, $2e^2\gamma/h$, where $\gamma$ is a constant different from $K_\rho$ [13]. This difference implies that taking into account only the screening effect due to forward scattering process is not sufficient for the correct treatment of the voltage drop. One needs to consider the renomalization of the external potential due to other scattering processes. The theoretical treatment of the voltage drop was also discussed by Egger and Grabert from another point of view based upon Landauer-type approach [20]. More recently, Kawabata pointed out that corrections to the conductance due to short-ranged electron-electron interaction may be absorbed into the renormalization of a chemical potential [21]. He demonstrated this in the case of backward scattering. By calculating corrections up to the first order in electron-electron interaction, he obtained the unrenormalized value of the conductance, $2e^2/h$. In this paper, we investigate the renormalization of the chemical potential due to Umklapp scattering in the case away from half-filling, which has not been considered in the previous studies. Our main purpose is to show that we have the unrenormalized value of the conductance, $2e^2/h$, by taking into account both the screening of external potential due to forward

* Dedicated to J. Zittartz on the occasion of his 60th birthday

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scattering and the renormalization of the chemical potential due to Umklapp scattering [22]. For this purpose, we need to calculate the second order processes of Umklapp scattering which give singular contributions. Thus, in contrast to the case of backward scattering, we cannot adopt a simple perturbation approach. We exploit bosonization and renormalization group method.

The organization of this paper is as follows. In Section 2, we define our model and summarize the results of our previous paper [13], in which only the screening of external potential due to forward scattering is considered. In Section 3, we develop perturbative renormalization group argument for the renormalization of the chemical potential due to Umklapp scattering. Combining the results in Sections 2 and 3, we show that the conductance is not renormalized even in the presence of Umklapp scattering in Section 4. In Section 5, we consider the generalization of our argument to fermion systems with SU(N) internal symmetry. This generalization is worthwhile to be investigated because of the following reason. It has been recently claimed by Zotos *et al.* that all 1D quantum *integrable* systems may have the properties of ideal conductors with zero-resistivity [23] even at finite temperatures. This assertion has been directly confirmed by the calculation based upon the Bethe ansatz solutions [24,25]. In this sense, integrable systems may exhibit rather special properties for transport coefficients in contrast to general non-integrable cases. Since our argument for the electron model (SU(2) case) is based on the bosonized effective hamiltonian, *i.e.* the 1D sine-Gordon model, which is integrable, we need to show that the unrenormalized conductance is not the consequence of the integrability, but a universal property inherent in 1D electron systems (even with Umklapp interaction). To confirm this point, we investigate a non-integrable SU(N) fermion model with Umklapp scattering. In the last section, some discussions about the applicability of our argument are given.

## 2 Screening of the external potential due to forward scattering

We consider an electron system with forward and Umklapp interactions, and start with the following Hamiltonian after linearizing the dispersion around the Fermi points,

$$\begin{aligned} H =& \mathrm{i}\hbar v_F \int dx \sum_\sigma : \psi^\dagger_{\sigma L}(x)\partial_x\psi_{\sigma L}(x) - \psi^\dagger_{\sigma R}(x)\partial_x\psi_{\sigma R}(x) : \\ &+ g\int \frac{dx}{2\pi} : \rho(x)\rho(x) : \\ &+ U\int \frac{dx}{2\pi} : e^{\mathrm{i}(4k_{\mathrm{F}}-2\pi)}\psi^\dagger_{\uparrow L}(x)\psi_{\uparrow R}(x)\psi^\dagger_{\downarrow L}(x)\psi_{\downarrow R}(x) + h.c. :, \end{aligned} \tag{1}$$

where :: represents the normal ordering, $\psi_{\sigma L(R)}$ is the annihilation operator for left(right)-moving electrons with spin $\sigma$, $g$ ($U$) is the coupling for forward (Umklapp) scattering, and $\rho(x) = \rho_L(x) + \rho_R(x)$ with $\rho_{L,(R)}(x) = \sum_\sigma \psi^\dagger_{\sigma L(R)}(x)\psi_{\sigma L(R)}(x)/\sqrt{2}$. According to Kawabata, [8,21] the renormalization of the external potential, $\Phi_0(q,\omega)$, occurs due to electron-electron interaction. In the present case, we have two kind of interactions, both of which may be expected to contribute to the potential renormalization. The forward scattering screens the external potential and changes the measured voltage. The Umklapp scattering gives rise to the non-uniform chemical potential shift in the presence of electric field gradient which contributes to the renormalization of external fields. The former effect was considered in our previous paper [13], which is briefly summarized in this section.

The screening effect can be incorporated by calculating the diagrams for current-current correlation function which are irreducible with respect to the forward scattering, $g$. Using bosonization method, we have the effective Hamiltonian for the charge degrees of freedom of equation (1),

$$H = H_0 + H_u, \tag{2}$$

$$H_0 = \int dx \left[ \frac{v_\rho}{2K_\rho}(\partial_x\phi_\rho(x))^2 + \frac{v_\rho K_\rho}{2}(\Pi_\rho(x))^2 \right], \tag{3}$$

$$H_u = \frac{U}{\alpha^2}\int \mathrm{d}x \cos(\sqrt{8\pi}\phi_\rho(x) + \delta x), \tag{4}$$

where $\alpha$ is the high-energy cut-off parameter. Here $\phi_\rho$ is a boson phase field for the charge degrees of freedom, $\Pi_\rho$ is its canonical conjugate field, and $\delta \equiv 4k_F - 2\pi$ with $k_F$ being the Fermi point. In the following, we will consider only the case away from half-filling, $\delta \neq 0$.

Since the Umklapp scattering term becomes irrelevant at the TL liquid fixed point, the leading correction to the conductance due to the Umklapp term can be estimated using perturbative calculations. The conductance is given by

$$\begin{aligned} G = \lim_{\omega\to 0} \frac{e^2\bar{\omega}^2}{L^2\omega} & \\ \times \int_{-L/2}^{L/2} dx \int_{-L/2}^{L/2} dx' \int_0^\beta d\tau \langle T\phi_\rho(x,\tau)\phi_\rho(x',0)\rangle & \\ \times e^{-i\bar{\omega}\tau}|_{\bar{\omega}=i\omega-\varepsilon}. & \end{aligned} \tag{5}$$

In order to take into account the renormalization of external potential, we should calculate the irreducible diagram with respect to the forward scattering, $g$, which is related to the charge susceptibility $\chi(q,\omega)$,

$$q^2\langle\phi_\rho(q,\omega)\phi_\rho(-q,\omega)\rangle^R_{irr} = \frac{\chi(q,\omega)}{1 - \frac{g}{2}\chi(q,\omega)}, \tag{6}$$

for $q \sim 0$. Here, $\langle\cdots\rangle^R$ is the retarded Green's function. We now expand $\chi(q,\omega)$ in terms of the strength of the Umklapp scattering $U$: $\chi(q,\omega) = \tilde{\chi}(q,\omega) + \delta\chi(q,\omega)$, where $\tilde{\chi}(q,\omega)$ includes only the effect of the forward scattering whereas $\delta\chi(q,\omega)$ is the correction due to the Umklapp

term. Then we have

$$q^2\langle\phi_\rho(q,\omega)\phi_\rho(-q,\omega)\rangle^R_{irr} = \frac{\tilde{\chi}(q,\omega)}{1-\frac{g}{2}\tilde{\chi}(q,\omega)} + \frac{\delta\chi(q,\omega)}{(1-\frac{g}{2}\tilde{\chi}(q,\omega))^2} = \chi_0(q,\omega) + \Big(\frac{\chi_0(q,\omega)}{\tilde{\chi}(q,\omega)}\Big)^2\delta\chi(q,\omega), \tag{7}$$

where $\chi_0(q,\omega)$ is the charge susceptibility for non-interacting electron systems. Evaluating $\delta\chi(q,\omega)$ up to the second order in $U$, we have the renormalized conductance [13],

$$G = \frac{2e^2}{h}(1-bU^2) \tag{8}$$

with $b = K_\rho(e^{(4-4K_\rho)l_c}-1)/(8-8K_\rho)$. Here $l_c$ is determined by the condition that $|4k_F - 2\pi| \sim 1/\alpha e^{l_c}$ where $\alpha$ is a high-energy cutoff [26]. Note again that the formula (8) is obtained by incorporating the screening effect due to forward scattering. This result should be modified, if one further takes into account properly the renormalization of the chemical potential due to Umklapp scattering, which will be discussed in the following sections.

## 3 Renormalization of the chemical potential due to Umklapp scattering

In this section, we discuss the renormalization of the non-uniform chemical potential due to the Umklapp scattering which results in the renormalization of the external electric fields. We use the perturbative renormalization group method. The coupling of charge currents to the external potential $\Phi(x,t)$ is given by

$$H_{ext} = -\sqrt{\frac{2}{\pi}}\int dx\Phi(x,t)\partial_x\phi_\rho(x). \tag{9}$$

Here, for simplicity and making our argument clear, we omit for a while the screening effect of external potential due to the forward scattering, and concentrate on the renormalization of the chemical potential due to the Umklapp scattering. Both effects are considered in the next section to obtain the final formula for the conductance.

Up to the first order in $\Phi(x,t)$ and the second order in $U$, we obtain the renormalization group equation for the external potential,

$$\frac{d\Phi(x,t)}{dl} = \Phi(x,t) - 2\pi\frac{U^2K_\rho}{v_\rho^2}J_0(\delta\alpha)\Phi(x,t) + 2\pi\frac{U^2K_\rho}{v_\rho^2}\frac{J_1(\delta\alpha)}{\delta\alpha}\Phi(x,t), \tag{10}$$

where $J_n(x)$ is the Bessel function.

Note that the renormalization equation for the uniform chemical potential is given by

$$\frac{d\delta}{dl} = \delta + 2\pi\frac{U^2K_\rho}{v_\rho^2}\frac{J_1(\delta\alpha)}{\alpha}, \tag{11}$$

where the quantity $\delta$ has been introduced in equation (4). Thus equation (10) is rewritten as,

$$\frac{d\Phi(x,t)}{dl} = \frac{1}{\delta}\frac{d\delta}{dl}\Phi(x,t) - 2\pi\frac{U^2K_\rho}{v_\rho^2}J_0(\delta\alpha)\Phi(x,t). \tag{12}$$

We impose the condition that $d\delta/dl = 0$ in order to conserve the electron density. Then we have

$$\frac{d\Phi(x,t)}{dl} = -2\pi\frac{U^2K_\rho}{v_\rho^2}J_0(\delta\alpha)\Phi(x,t). \tag{13}$$

The current induced by the bare external potential $\Phi_0(x,t)$ at zero temperature is $I = (2e^2/h)K_\rho\Delta\Phi_0$ with $\Delta\Phi_0 = \Phi_0(+\infty,t) - \Phi_0(-\infty,t)$. Thus the conductance is given by

$$G = \frac{2e^2}{h}\frac{K_\rho\Delta\Phi_0}{\Delta\Phi}\,. \tag{14}$$

Using equation (13) and the renormalization equation of $K_\rho$ [26],

$$\frac{dK_\rho}{dl} = -2\pi\frac{U^2K_\rho^2}{v_\rho^2}J_0(\delta\alpha), \tag{15}$$

we have,

$$\frac{dG}{dl} = 0. \tag{16}$$

Thus the conductance is not renormalized by the Umklapp scattering for any scaling length at zero temperature. This remarkable property provides the basis for the following argument about the universal conductance in the presence of the Umklapp scattering. To avoid confusions, we wish to mention again that the formula (16) has been obtained by omitting the potential-renormalization effect due to forward scattering for simplicity.

## 4 Universal conductance in the presence of Umklapp scattering

Now we are ready to show that the conductance is not renormalized even in the presence of Umklapp scattering if one incorporates both effects of the screening of external potential due to forward scattering and the renormalization of the chemical potential due to Umklapp scattering.

The averaged value of the current in the static limit is given by

$$\langle J(x)\rangle = \lim_{\omega\to0}\int dq\frac{1}{q}\chi(q,\omega)\Phi_0e^{iqx} = \lim_{\omega\to0}\int dq\frac{1}{q}\tilde{\chi}(q,\omega)\Phi_0(q,\omega)e^{iqx} + \frac{2e^2}{h}\delta K_\rho\Delta\Phi_0, \equiv J_1(x) + J_2(x), \tag{17}$$

where we have expanded the charge susceptibility $\chi(q,\omega)$

in terms of $U$, and separated the current into two parts: $J_1(x)$, which includes only the effect of the forward scattering, and $J_2(x) = (2e^2/h)\delta K_\rho \Delta\Phi_0$, the correction due to the Umklapp scattering. This expansion is justified for small $U$ since the Umklapp interaction is an irrelevant operator. We also expand the external field in terms of $U$,

$$\Phi'(x,t) = \Phi(x,t) + \delta\Phi(x,t), \tag{18}$$

where $\Phi(x,t)$ includes only the screening effect due to the forward scattering, and $\delta\Phi(x,t)$ is the correction due to the Umklapp scattering discussed in Section 3.

Then the contribution to the conductance from the first term of the last line in equation (17) is given by up to the lowest order in $U$,

$$G_1 \equiv \frac{\int dx J_1(x)}{\Delta\Phi'} = \frac{2e^2}{h} - \frac{2e^2}{h}\tilde{K}_\rho \frac{\Delta\Phi_0 \Delta(\delta\Phi)}{(\Delta\Phi)^2} + O(U^3). \tag{19}$$

Here $\tilde{K}_\rho$ includes only the contribution from forward scattering, and $\Delta(\delta\Phi) = \delta\Phi(+\infty) - \delta\Phi(-\infty)$. In order to obtain this expression, we have used the relation $\chi_0(q,\omega)\Phi = \tilde{\chi}(q,\omega)\Phi_0$ [8,13]. The contribution to the conductance from $J_2(x)$ up to the same order in $U$ is given by,

$$G_2 \equiv \frac{\int dx J_2(x)}{\Delta\Phi'} \simeq \frac{2e^2}{h}\frac{\delta K_\rho \Delta\Phi_0}{\Delta\Phi}\,. \tag{20}$$

The corrections due to Umklapp scattering, $\Delta(\delta\Phi)$ and $\delta K_\rho$, are evaluated by using the renormalization group equation obtained in Section 3,

$$\Delta(\delta\Phi) = \int dl \frac{d\Delta\Phi}{dl} = -2\pi \int dl \frac{K_\rho U^2}{v_\rho^2} J_0(\delta\alpha)\Delta\Phi, \tag{21}$$

$$\delta K_\rho = -2\pi \int dl \frac{K_\rho^2 U^2}{v_\rho^2} J_0(\delta\alpha). \tag{22}$$

If we consider the lowest order corrections in $U$, $K_\rho$ in the right-hand side of equations (21, 22) is replaced by $\tilde{K}_\rho$, and $\Delta\Phi$ in the right-hand side of equation (21) does not depend on the scaling parameter $l$. Then from equations (19, 20), we have the conductance,

$$G = G_1 + G_2 = \frac{2e^2}{h}\,. \tag{23}$$

Therefore we come to the conclusion that the conductance is not renormalized even in the presence of Umklapp scattering if one takes into account not only the screening of the external potential but also the renormalization of the local chemical potential. This conclusion, which may improve our previous results [13], is in accordance with the recent studies on the effect of non-interacting leads attached to quantum wires in the presence of Umklapp scattering [17,18].

## 5 Generalization to the SU(N) fermion model

As mentioned in the introduction, the model we considered in the previous sections, the sine-Gordon model, is an integrable system, in which it is known that no true current decay occurs [23]. Thus one may suspect that the unrenormalized conductance obtained in the previous sections might be the consequence of the integrability of the model. In this section, in order to examine this point, we consider the non-integrable 1D interacting electron systems of which the spin degrees of freedom is generalized to the SU(N) symmetry. In this model, the bosonized form of the Umklapp interaction which breaks the integrability is given by [27],

$$H_{umklapp} = \frac{U}{\alpha^2}\int dx (g_\beta^\alpha g_\alpha^\beta - g_\alpha^\alpha g_\beta^\beta)\exp(i\sqrt{16\pi/N}\phi_\rho + \delta x) + h.c., \tag{24}$$

where $g_\beta^\alpha$ is a matrix of SU(N) Lie group. As seen from the above interaction, the charge degrees of freedom is coupled with SU(N) internal degrees of freedom. In the case away from half-filling $\delta \neq 0$, this term is irrelevant, and can be treated perturbatively. Following the method in Section 3, we obtain the scaling equations for external potential and the Luttinger parameter $K_\rho$,

$$\frac{d\Phi(x,t)}{dl} = -4\pi\frac{U^2 K_\rho}{N v_\rho^2} J_0(\delta\alpha)\Phi(x,t), \tag{25}$$

$$\frac{dK_\rho}{dl} = -4\pi\frac{U^2 K_\rho^2}{N v_\rho^2} J_0(\delta\alpha). \tag{26}$$

Using these equations, we can repeat the same argument as done for the SU(2) case, and obtain the unrenormalized value of the conductance, $2e^2/h$. Thus, our result is not restricted to integrable systems. Although our argument is based upon a specific model, we believe that the result for the unrenormalized conductance in the presence of Umklapp scattering is a universal property of 1D metallic systems.

## 6 Discussions

In this paper, we have shown that the conductance takes universal value, $2e^2/h$, in the presence of Umklapp scattering, by properly taking into account not only the screening of external potential due to forward scattering of electron-electron interaction but also the renormalization of chemical potential due to Umklapp scattering. The conclusion is in accordance with the theoretical result obtained for the system with non-interacting leads being attached to a quantum wire, although the mechanism to obtain the universal value is different between two approaches. Here we discuss about the condition in which our argument is applicable. The renormalization of the local chemical potential stems from the local charge density fluctuation induced by the external potential. In the derivation of equation (10), we assumed that the induced charge density, $\chi_c\Phi(x,t) = (2K_\rho/\pi v_\rho)\Phi(x,t)$ is much smaller than the total charge density $n$. This is nothing but the condition required for the applicability of linear response theory. As the electron density approaches half-filling

$n \to 1$, the charge susceptibility $\chi_c$ diverges [28] like $\sim 1/(1-n)$. Thus in the vicinity of the Mott transition, the value of $\Phi(x,t)$ for which the unrenormalized conductance is observed is quite small, $\Phi(x,t) \ll n/\chi_c$. We cannot apply Landauer's formula unless this condition is satisfied. Thus although the conductance is not renormalized for any electron densities away from half-filling, the range of the applied external potential for which the unrenormalized conductance is observable becomes smaller, as the electron density approaches the half-filling. A sufficiently large external potential may excite electrons to the upper-Hubbard band. In this case, non-linear effects become very important. It may be an interesting issue to investigate such non-linear effects which characterize the precursor of the Mott transition.

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